

1.  $y = x^2$

Homogeneity:

$$(\alpha x)^2 = \alpha^2 x^2 \neq \alpha y$$

Additivity:

$$(x_1 + x_2)^2 = x_1^2 + x_2^2 + 2x_1x_2 \neq y_1 + y_2$$

The system is Non-Linear

2.  $y = \frac{dx}{dy}$

Homogeneity:

$$\frac{d(\alpha x)}{dt} = \alpha \frac{dx}{dy} = \alpha y$$

Additivity:

$$\frac{d(x_1 + x_2)}{dt} = \frac{dx_1}{dt} + \frac{dx_2}{dt} = y_1 + y_2$$

The system is linear

3.  $\frac{dy}{dt} = ay + x$

Homogeneity:

$$\frac{d(\alpha y)}{dy} = a \alpha y + \alpha x = \alpha \frac{dy}{dy} = \alpha (ay + x)$$

Additivity:

$$\frac{d(y_1 + y_2)}{dt} - a(y_1 + y_2) = \frac{dy_1}{dt} + \frac{dy_2}{dt} - ay_1 - ay_2 = x_1 + x_2$$

4.  $y = mx + b$

Homogeneity:

$$m(\alpha x) + b = \alpha(mx) + b \neq \alpha y$$

Additivity:

$$m(x_1 + x_2) + b = mx_1 + mx_2 + b \neq y_1 + y_2$$

The systems is non-linear